ARO-D-968.62-M

FINAL REPORT

"Fundamental Investigations in Operations Research"

U. S. Army Research Office - Durham
ARO-D Project Number 968-M
Contract DA-31-124-ARO-D-209
M.I.T. DSR 75217

Contract Period: July 1, 1964 - September 30, 1970

Operations Research Center
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Author of Report: John D. C. Little

This final report covers the contract period, July 1, 1964 through September 30, 1970. Professor Philip M. Morse, Director of the Operations Research Center, was Principal Investigator from July 1, 1964 through June 30, 1967. Joining him as Co-Principal Investigator from July 1, 1967 through September 30, 1970 was Professor Alvin W. Drake, Associate Director of the Operations Research Center. Professor Morse retired June 30, 1969, but continues at the Center part time. Professor John D. C. Little was appointed Director July 1, 1969; Professor Drake continues as Associate Director.

STATEMENT OF PROBLEM STUDIED

This project has been concerned with extending the fundamental theories underlying operations research. In particular it has focused on certain areas in mathematical programming, adaptive control, decision theory, and queuing theory. The research in mathematical programming has centered on integer and mixed integer problems. The adaptive control work has dealt with special classes of stochastic decision problems. The multidimensional utility function problem has been studied in decision theory. In queuing theory, which has long been descriptive in its approach, the emphasis has been on normative models, especially Markovian decision processes.

PUBLICATIONS supported entirely or in part by ARO-D during the contract period:

Journal Articles and Books

Morgan, J. T. and J. D. C. Little, "Synchronizing Traffic Signals for Maximal Bandwidth," Operations Research, 6, 896-912 (November, 1964).

Little, J. D. C., B. V. Martin, and J. T. Morgan, "Synchronizing Traffic Signals for Maximal Bandwidth," Highway Research Record No. 118, 1966.

Little, J. D. C., "The Synchronization of Traffic Signals by Mixed-Integer Linear Programming," Operations Research, 14, 568-594 (1966).

Little, J. D. C., "A Model of Adaptive Control of Promotional Spending," Operations Research, 14, 1075-1097 (November, 1966).

Holland, Homer J., "A Stochastic Model for Multilane Traffic Flow," Transportation Science, 1, No. 3, 184-205 (August, 1967).

Publications (cont'd)

Miller, S. B. and J. D. C. Little, "Evaluation and Improvement of Traffic Signal Settings by Simulation," Highway Research Record No. 170, 56-69 (1967).

Hurst, Jr., E. G. and A. B. McNamara, "Heuristic Scheduling in a Woolen Mill," Management Science, 14, No. 4, B-182-203 (December, 1967).

Shapiro, J. F., "Dynamic Programming Algorithms for the Integer Programming Problem - I: The Integer Programming Problem Viewed as a Knapsack Type Problem," Operations Research, 16, 103-121 (January-February, 1968).

Dixit, A. K., "Optimal Development in the Labour-Surplus Economy," Review of Economic Studies, 35, No. 1, 23-24 (1968).

Obregon, Ivan, "Optimal Inventory Levels for Slowly Moving Items," (in Spanish), DELTA, 1, No. 1, 9-16 (August, 1968).

Hurst, Jr., E. G., "Bayesian Autoregressive Time Series Analysis," IEEE Transactions on Systems Science and Cybernetics, SSC-4, No. 3, 317-324 (September, 1968).

Shapiro, J. F., "Group Theoretic Algorithms for the Integer Programming Problem - II: Extension to a General Algorithm," Operations Research, 16, No. 5, 928-947 (September-October, 1968).

Shapiro, J. F., "Shortest Route Methods for Finite State Space Deterministic Dynamic Programming Problems," SIAM Journal of Applied Mathematics, 16, No. 6, 1232-1250 (November, 1968).

Morse, P. M., Library Effectiveness: A Systems Approach, The M.I.T. Press, 1968. (Cowinner of the Lanchester Prize for 1968, awarded by the Operations Research Society of America.)

Keeney, R. L., "Quasi-Separable Utility Functions," Naval Research Logistics Quarterly, 15, No. 4, 551-565 (December, 1968).

Wilde, N. W. G., "Differential Equation Approach to Nonlinear Programming," Management Science, 15, 739 (1969).

Morse, P. M. and C. Elston, "A Probabilistic Model for Obsolescence," Operations Research, 17, 36-47 (January-February, 1969).

Odoni, A. R., "On Finding the Maximal Gain for Markov Decision Processes," Operations Research, 17, No. 5, 857-860 (September-October, 1969).

Publications (cont'd)

Obregon, I. and L. J. Mira, "Optimal Plant Location Through Dynamic Programming," (in Spanish), DELTA, 1, No. 2, 46-55 (March, 1969).

Keeney, R. L., "On the Trinomial Coefficients," Mathematics Magazine, 42, 210-212 (September, 1969).

Obregon, I., "Linear Programming," (in Spanish), DELTA, 1, No. 4, 3-20 (December, 1969).

Little, J. D. C., "Managers and Models: The Concept of a Decision Calculus," Management Science, 16, B466-485 (April, 1970).

Shapiro, J. F., "Turnpike Theorems for Integer Programming Problems," Operations Research, 18, No. 3, 432-440 (May-June, 1970).

Keeney, R. L., "Assessment of Multiattribute Preferences," Science, 168, 1491-1492 (June 19, 1970).

In Process

Gorry, G. A. and J. F. Shapiro, "An Adaptive Group Theoretic Algorithm for Integer Programming Problems," to be published in Management Science (issue of January, 1971).

Shapiro, J. F., "Generalized Lagrange Multipliers in Integer Programming," to be published in Operations Research (issue of March-April, 1971).

Eisenberg, M., "Two Queues with Changeover Times," to be published in Operations Research.

Keeney, R. L., "Utility Independence and Preferences for Multiattributed Consequences," to be published in Operations Research.

Morse, P. M. and H. J. Yaffe, "A Queuing Model for Car Passing," to be published in Transportation Science.

Welsch, R. E., "Limit Laws for Extreme Order Statistics from Strong-Mixing Processes," to be published in Annals of Mathematical Statistics.

Gorry, G. A., J. F. Shapiro, and L. A. Wolsey, "Relaxation Methods for Pure and Mixed Integer Programming Problems," submitted to Management Science.

Publications (cont'd)

Garry, G. A., G. L. Nemhauser, W. D. Northrup, and J. F. Shapiro, "An Improved Branching Rule for the Group Theoretic Branch-and-Bound Integer Programming Algorithm," submitted to Operations Research.

Keeney, R. L., "Risk Independence and Multiattributed Utility Functions," submitted to Econometrica.

Keeney, R. L., "Utility Functions for Multiattributed Consequences," submitted to Management Science.

Technical Reports and Working Papers

Patel, N. R., "A Mathematical Analysis of Computer Time-Sharing Systems," M.I.T. Operations Research Center Technical Report No. 8, July, 1964.

Wheda, P. J., "The Stochastic Behavior of Traffic on a Two-Lane Road," M.I.T. Operations Research Center Technical Report No. 9, November, 1964.

Murray, G. R., and K. R. Scott, "Demand Models for Inventory Systems," M.I.T. Operations Research Center Technical Note No. 4, March, 1965.

Hurst, E. G., "A Dynamic Programming Formulation for Disposable and Reparable Inventory," M.I.T. Operations Research Center Technical Report No. 14, June, 1965.

Gonzalez, R. H., "On Some Aspects of Integer Linear Programming," M.I.T. Operations Research Center Technical Report No. 16, June, 1965.

Kakalik, J. S., "Optimum Policies for Partially Observable Markov Systems," M.I.T. Operations Research Center Technical Report No. 18, October, 1965.

Little, J. D. C., "The Synchronization of Traffic Signals by Mixed-Integer Linear Programming," M.I.T. Operations Research Center Technical Report No. 19, December, 1965.

Miller, S. B. and J. D. C. Little, "The Evaluation and Improvement of Traffic Signal Settings by Simulation," M.I.T. Operations Research Center Technical Report No. 22, November, 1966.

Oliregon, I., "On the N-Armed Bandit and Other Problems Involving Sequential Decisions," M.I.T. Operations Research Center Technical Report No. 24, November, 1966.

Publications (cont'd)

Odoni, A. R., "Alternative Schemes for Investigating Markov Decision Processes," M.I.T. Operations Research Center Technical Report No. 28, June, 1967.

Holland, H. J., "A Stochastic Model for Multi-Traffic Flow," M.I.T. Operations Research Center Technical Report No. 25, June, 1967.

Hurst, Jr., E. G., "A Class of Models for Adaptive Experimentation and Control," Technical Report No. 30, M.I.T. Operations Research Center, July, 1967.

Shapiro, J. F., "Shortest Route Methods for Finite State Space, Deterministic Dynamic Programming Problems," Technical Report No. 31, M.I.T. Operations Research Center, July, 1967.

Raynaud, T. G., "Operational Analysis of a Computation Center," Technical Report No. 32, M.I.T. Operations Research Center, July, 1967.

Keeney, R. L., "Quasi-Separable Utility Functions," Technical Report No. 33, M.I.T. Operations Research Center, December, 1967.

Eisenberg, M., "Multi-Queues with Changeover Times," Technical Report No. 35, M.I.T. Operations Research Center, February, 1968.

Shapiro, J. F., "Group Theoretic Algorithms for the Integer Programming Problem II: Extension to a General Algorithm," Technical Report No. 38, M.I.T. Operations Research Center, March, 1968.

Shapiro, J. F., "Turnpike Theorems for Integer Programming Problems," Working Paper 350-68, M.I.T. Sloan School of Management.

Gorry, G. A. and J. F. Shapiro, "An Adaptive Group Theoretic Algorithm for Integer Programming Problems," Technical Report No. 39, M.I.T. Operations Research Center, May, 1968; revised May, 1969.

Wolsey, L. A., "Group Representational Theory in Integer Programming," Technical Report No. 41, M.I.T. Operations Research Center, February, 1969.

Shapiro, J. F., "Generalized Lagrange Multipliers in Integer Programming," Working Paper 371-69, M.I.T. Sloan School of Management (1969).

Wolsey, L. A., "Mixed Integer Programming: Discretization and the Group Theoretic Approach," Technical Report No. 42, M.I.T. Operations Research Center, June, 1969.

Publications (cont'd)

Chandy, K. Mani, "Parametric Decomposition Programming," M.I.T. Operations Research Center Technical Report No. 45, October, 1969.

Kakalik, James S., "Optimal Dynamic Operating Policies for a Service Facility," M.I.T. Operations Research Center Technical Report No. 47, October, 1969.

Keeney, Ralph L., "Multidimensional Utility Functions: Theory, Assessment, and Application," M.I.T. Operations Research Center Technical Report No. 43, October, 1969.

Hamilton, Carl W., "Optimal Control of Research and Development Expenditures," M.I.T. Operations Research Center Technical Report No. 48, December, 1969.

Shapiro, Jeremy F., "Generalized Lagrange Multipliers in Integer Programming," Working Paper 371-69, M.I.T. Sloan School of Management (1969).

Odoni, Amedeo R., "An Analytical Investigation of Air Traffic in the Vicinity of Terminal Areas," M.I.T. Operations Research Center Technical Report No. 46, December, 1969.

Tavernier, H. M., "Optimization of Linear Discrete Systems: An Approach to the Staircase Problem," Working Paper 459-70, M.I.T. Sloan School of Management, April, 1970.

Morse, Philip M., "On Browsing: The Use of Search Theory in the Search for Information," M.I.T. Operations Research Center Technical Report No. 50, February, 1970.

Morse, Philip M., and Harold J. Yaffe, "A Queuing Model for Car Passing," M.I.T. Operations Research Center Technical Report No. 51, April, 1970.

Gorry, G. A., J. F. Shapiro, and L. A. Wolsey, "Relaxation Methods for Pure and Mixed Integer Programming Problems," Working Paper 456-70, M.I.T. Sloan School of Management, April, 1970.

Welsch, Roy E., "Limit Laws for Extreme Order Statistics from Strong-Mixing Processes," M.I.T. Operations Research Center Technical Report No. 52, April, 1970.

Mehta, C. R., "Time Sharing with an Explicit Priority Queuing Discipline," M.I.T. Operations Research Center Technical Report No. 54, July, 1970.

Publications (cont'd)

Crowston, W. B., M. H. Wagner, and A. Henshaw, "A Comparison of Exact and Heuristic Routines for Lot Size Determination in Multi-State Assembly Systems," Working Paper 475-70, M.I.T. Sloan School of Management, September, 1970.

Fisher, Marshall L., "Optimal Solution of Resource Constrained Network Scheduling Problems," M.I.T. Operations Research Center Technical Report No. 56, September, 1970.

Wagner, M. H., "Multi-Stage Assembly Systems," M.I.T. Sloan School of Management Working Paper (forthcoming).

SCIENTIFIC PERSONNEL supported entirely or in part by ARO-D during the contract period:

Prof. Philip M. Morse
July 1, 1964 - August 31, 1964
Sept. 16, 1964 - June 15, 1965
July 11, 1965 - August 31, 1965
Sept. 1-10 & 16-30, 1965 - June 15, 1966
Sept. 16, 1966 - June 15, 1967
Sept. 16, 1967 - June 15, 1968
Sept. 16, 1968 - June 15, 1970

Prof. E. Farnsworth Bisbee
Oct. 16, 1964 - June 15, 1965

Dr. George M. Murray
Jan. 1, 1965 - July 31, 1965

Prof. Jeremy F. Shapiro
June 16, 1967 - June 15, 1968
Sept. 16, 1968 - Sept. 30, 1969

Prof. Alvin W. Drake
Sept. 1, 1967 - Jan. 31, 1968
Feb. 1, 1969 - June 15, 1969

Prof. John D. C. Little
Sept. 16, 1969 - June 15, 1970
Sept. 1, 1970 - Sept. 30, 1970

Scientific Personnel (cont'd)

Prof. Roy E. Welsch July 1, 1970 - July 31, 1970		
Harold D. Cluck Sept. 16, 1964 - Jan. 31, 1965	E.E.	February 1965
E. Gerald Hurst, Jr. Sept. 16, 1964 - Jan. 31, 1965	S.M. Ph.D.	June 1965 June 1967
Romulo H. Gonzalez Sept. 16, 1964 - June 15, 1965	Ph.D.	June 1965
Terence E. Daniel Sept. 16, 1965 - Oct. 31, 1965	S.M.	June 1967
Ivan Obregon Sept. 16, 1965 - Sept. 15, 1966	Ph.D.	September 1966
Avinash Dixit Sept. 16, 1965 - June 15, 1966 Sept. 16, 1966 - June 15, 1967	Ph.D.	June 1968
Hans A. Herriger Sept. 16, 1966 - Jan. 31, 1967	S.M.	February 1967
Martin Eisenberg June 16, 1967 - Sept. 15, 1967	Ph.D.	September 1967
John D. Steinbruner Sept. 16, 1967 - Jan. 31, 1968	Ph.D.	February 1968
Donald E. Lewin Sept. 16, 1967 - June 15, 1968	(Ph.D. expected Feb. 1971)	
Joel Shwimer Sept. 16, 1967 - June 15, 1968	(Ph.D. expected Feb. 1972)	
Laurence A. Wolsey Sept. 16, 1967 - June 15, 1968 Sept. 16, 1968 - June 15, 1969	Ph.D.	June 1969
Norman W. G. Wilde Feb. 1, 1969 - June 15, 1969 Sept. 16, 1969 - June 15, 1970	(Ph.D. expected Feb. 1971)	

Scientific Personnel (cont'd)

Thomas K. Zaslavsky
Sept. 16, 1969 - Jan. 31, 1970 (Ph. D. expected Feb. 1971)

Hubert M. Tavernier
Sept. 16, 1969 - June 15, 1970 (Ph. D. expected Feb. 1971)

Michael H. Wagner
Sept. 16, 1969 - June 15, 1970 (Ph. D. expected June 1971)

DEGREES AWARDED during contract period, supported (in part) by ARO-D:

June 1964

Patel, Nitin R., "A Mathematical Analysis of Computer Time-Sharing Systems," S. M. Thesis, Dept. of Electrical Engineering, supervised by Greenberger.

Weeda, Pieter J., "The Stochastic Behavior of Traffic on a Two-Lane Road," S. M. Thesis, Dept. of Physics, supervised by Morse.

June 1965

Gonzalez, Romulo H., "On Some Aspects of Integer Linear Programming," Ph. D. Thesis, Sloan School of Management, supervised by Little.

Hurst, Jr., E. Gerald, "A Dynamic Programming Formulation for Disposable and Reparable Inventory," S. M. Thesis, Dept. of Civil Engineering, supervised by Bisbee.

September 1965

Kakalik, James S., "Optimum Policies for Partially Observable Markov Systems," S. M. Thesis, Dept. of Electrical Engineering, supervised by Drake.

September 1966

Obregon, Ivan, "On the N-Armed Bandit and Other Problems Involving Sequential Decisions," Ph. D. Thesis, Dept. of Civil Engineering, supervised by Little.

Degrees Awarded (cont'd)

February 1967

Herriger, Hans A., "The Effect of Internal Buffer Storage Upon the Efficiency of Automated Assembly Lines," S. M. Thesis, Sloan School of Management, supervised by White.

Holland, Homer J., "A Stochastic Model for Traffic Flow on a Two-Lane One-Way Highway," S. M. Thesis, Dept. of Physics, supervised by Morse.

Odoni, Amedeo R., "Alternative Schemes for Investigating Markov Decision Processes," S. M. Thesis, Dept. of Electrical Engineering, supervised by Drake.

June 1967

Hurst, Jr., E. Gerald, "A Class of Models for Adaptive Experimentation and Control," Ph. D. Thesis, Sloan School of Management, supervised by Little.

Raynaud, Thierry, "Operational Evaluation of a Computation Center," S. M. Thesis, Dept. of Electrical Engineering, supervised by Morse.

September 1967

Eisenberg, Martin, "Multi-Queuing Systems with Changeover Times," Ph. D. Thesis, Dept. of Electrical Engineering, supervised by Drake.

Keeney, Ralph L., "Quasi-Separable Utility Functions," S. M. Thesis, Dept. of Electrical Engineering, supervised by Drake.

February 1968

Steinbruner, John D., "A Psychological Study of Decision-Making Processes--The American MLF Proposal," Ph. D. Thesis, Dept. of Political Science, supervised by W. Kaufman.

Zacks, Leonard H., "An Analysis of Arrival Policies for Parallel Channel Queuing Facilities," S. M. Thesis, Dept. of Physics, supervised by Morse.

Degrees Awarded (cont'd)

June 1968

Dixit, Avinash, "Development Planning in a Dual Economy," Ph. D. Thesis, Dept. of Economics, supervised by Solow.

Ingram, William A., "Investigation of the Effects of Using Different Levels of Accessibility in a Library System," S. B. Thesis, Dept. of Physics, supervised by Morse.

February 1969

Fisher, Marshall, "Branch and Bound Algorithm for Optimizing a Function Defined by A Simulation Model," S. M. Thesis, Sloan School of Management, supervised by Carroll.

June 1969

Keeney, Ralph L., "Multidimensional Utility Functions: Theory, Assessment, and Application," Ph. D. Thesis, Interdepartmental, supervised by Raiffa.

Wolsey, Laurence A., "Mixed Integer Programming: Discretization and the Group Theoretic Approach," Ph. D. Thesis, Dept. of Mathematics, supervised by Shapiro.

September 1969

Chandy, K. Mani, "Parametric Decomposition Programming," Ph. D. Thesis, Dept. of Electrical Engineering, supervised by Shapiro.

Hamilton, Carl W., "Optimal Control of Research and Development Expenditures," Ph. D. Thesis, Sloan School of Management, supervised by Shapiro.

Kakalik, James S., "Optimal Dynamic Operating Policies for a Service Facility," Ph. D. Thesis, Interdepartmental, supervised by Little.

Odoni, Amedeo R., "An Analytical Investigation of Air Traffic in the Vicinity of Terminal Areas," Ph. D. Thesis, Dept. of Electrical Engineering, supervised by Simpson.

Degrees Awarded (cont'd)

February 1970

Mehta, Cyrus R., "Time Sharing with an Explicit Priority Queuing Discipline," S. M. Thesis, Sloan School of Management, supervised by Kaufman.

Zacks, Leonard H., "A Queueing Theoretic Analysis of Contractors' Sequential Bidding Problems," Ph. D. Thesis, Interdepartmental, supervised by Kaufman.

June 1970

Papademos, Lucas D., "Markov Decision Processes and the Analysis of a Queuing System," S. B. Thesis, Dept. of Physics, supervised by Morse.

Yaffe, Harold J., "The Queuing of Traffic on a Country Road," S. B. Thesis, Dept. of Physics, supervised by Morse.

September 1970

Fisher, Marshall L., "Optimal Solution of Resource Constrained Network Scheduling Problems," Ph. D. Thesis, Sloan School of Management, supervised by Shapiro.

SUMMARY OF RESEARCH FINDINGS NOT PREVIOUSLY REPORTED

Mathematical Programming

Fisher, Marshall L., "Optimal Solution of Resource Constrained Network Scheduling Problems," M.I.T. Operations Research Center Technical Report No. 56, September, 1970. Adapted from a Ph. D. thesis, Sloan School of Management, September, 1970, supervised by Prof. J. F. Shapiro.

A class of resource constrained network scheduling problems is defined in which a set of start times for tasks must be determined which minimize some function of the task completion times. Included in this class is the job shop scheduling problem. Three major ideas for the optimal solution of these problems are presented.

First, the special case in which resources are available and used in unit amounts is formulated as a 0-1 IP problem in which all coefficients are 0 or 1 and all right-hand sides are 1. It is shown that every edge of the

Summary of Research Findings (cont'd)

integer polyhedron for this problem is also an edge of the continuous polyhedron and the implications for applying the simplex method are developed.

In the second major idea, we generalized the 0-1 IP formulation previously developed to include the entire class of scheduling problems which have been defined. The formulation is used to derive a family of lower bounds on the optimal objective value for a particular scheduling problem. A particularly strong bound is incorporated in a tree search algorithm for solving the general scheduling problem.

The third idea is that a scheduling problem may be modified by dividing all task process times by a constant $\lambda > 1$ and rounding down to the nearest integer to obtain a new problem. The optimal objective value of the new problem, multiplied by λ, is a lower bound on the optimal objective value for the original problem.

Tavernier, H. M., "Optimization of Linear Discrete Systems: An Approach to the Staircase Problem," Working Paper 459-70, M.I.T. Sloan School of Management, April, 1970.

The optimization of a stationary staircase problem is studied with an approach that mixes concepts of mathematical programming and optimal control. Some results are first derived about feasibility and controllability. Then optimization is approached by dynamic programming to show that the problem can be reformulated as a decoupled optimization problem. Suggestions are made to take advantage of this fact in designing practical algorithms.

Shapiro, Jeremy F., "Turnpike Theorems for Integer Programming Problems," Operations Research, 18, No. 3, 432-440 (May-June, 1970).

A certain class of integer programming problems, called asymptotic or steady-state, has been shown by Gomory to be cost equivalent to a group-optimization problem. This paper extends the algebraic characterization by demonstrating that there are cost-equivalent group problems for all integer programming problems. Finally, the result is interpreted from the viewpoint of dynamic programming, and this provides the turnpike theorem.

Summary of Research Findings (cont'd)

Gorry, G. Anthony, and Jeremy F. Shapiro, "An Adaptive Group Theoretic Algorithm for Integer Programming Problems," to be published in Management Science (issue of January, 1971).

Group theory is used to integrate a wide variety of integer programming methods into a common computational process. Included are group optimization algorithms, Lagrangian methods, the cutting plane method, and the method of surrogate constraints. These methods are controlled by a supervisor which performs four main functions: set-up, directed search, subproblem analysis, and prognosis.

Some computational experience is given. An appendix contains an algorithm for dynamically solving unconstrained group problems. A second appendix gives an algorithm for solving zero-one group problems.

Shapiro, Jeremy F., "Generalized Lagrange Multipliers in Integer Programming," to be published in Operations Research (issue of March-April, 1971).

In this paper, the theory of generalized Lagrange multipliers is combined with a reformulation of the integer programming problem due to group theory. The use of multipliers enhances the algorithmic efficiency of group theory in a variety of ways. One particular application is the approximation of generalized Lagrange multipliers by generalized linear programming as suggested by Brooks and Geoffrion. This procedure is shown to be closely related to the cutting plane method of Gomory.

Gorry, G. A., J. F. Shapiro, and L. A. Wolsey, "Relaxation Methods for Pure and Mixed Integer Programming Problems," submitted to Management Science.

The usefulness of group theoretic methods in solving integer programming (IP) problems is extended by procedures for controlling the size of the groups. The main procedure shows how an optimal linear programming basis can be altered to reduce the magnitude of its determinant, thereby reducing the size of the group induced by the basis. An adaptation of Bender's mixed IP algorithm is given which uses these methods. Some limited computational experience is given.

Summary of Research Findings (cont'd)

Gorry, G. Anthony, George L. Nemhauser, William D. Northrup, and Jeremy F. Shapiro, "An Improved Branching Rule for the Group Theoretic Branch-and-Bound Integer Programming Algorithm," submitted to Operations Research.

A previous paper contains a branch and bound integer programming (IP) algorithm which exploits a group optimization problem identified by Gomory. This note presents an improved branching rule which (1) eliminates certain computation that is useless, and (2) serves to intensify the search for an optimal solution in an area of promising solutions.

Crowston, W. B., M. H. Wagner, and A. Henshaw, "A Comparison of Exact and Heuristic Routines for Lot Size Determination in Multi-Stage Assembly Systems," Working Paper 475-70, M.I.T. Sloan School of Management, September, 1970.

The Wilson, or classical, economic lot size model is generalized to multi-stage assembly systems in which lot sizes need to be determined for constituent parts and subassemblies as well as for final products. Under the usual assumption of constant demand over an infinite horizon, a characterization of the form of the optimal solution is obtained. Dynamic programming is then used to find optimal solutions, and comparisons are made with results obtained with various heuristics.

Tavernier, H. M., "Constrained Optimization of Discrete Dynamic Systems," Ph. D. thesis in progress under the supervision of Prof. J. F. Shapiro.

Many linear, quadratic, or convex programming problems should be viewed in a dynamic environment. This is particularly true for planning problems. The present research is designed to apply some of the concepts of a system theory and optimal control to the programming of discrete time, state constrained, dynamic systems of the staircase type.

Wilde, Norman W. G., "Numerical Analysis of Dynamic Programming," Ph. D. thesis in progress under the supervision of Prof. J. F. Shapiro.

This study analyzes the numerical properties of dynamic programming algorithms, emphasizing the effects of approximation procedures on the accuracy of the result. Error propagation formulas are developed and sufficient conditions are provided for the convergence of approximate dynamic programs to the true solution. A "state reduction" dynamic programming algorithm for the deterministic case is proposed which offers the possibility of perfect convergence over continuous state spaces using only a finite amount of computer memory.

Summary of Research Findings (cont'd)

Wagner, Michael H., "Multi-Stage Assembly Systems," M.I.T. Sloan School of Management Working Paper (forthcoming).

Lot size determination for multi-stage assembly systems is considered for both a finite horizon model with varying, but known, demands, and for infinite horizon models with constant demands. Characterizations of the form of optimal solutions are developed under the assumption of concave, time-invariant, production and inventory cost functions. Heuristics and a dynamic programming algorithm for the infinite horizon cases are described and computational results reported. Solution of the finite-horizon case by dynamic programming and by Bender's method for mixed integer programming is discussed.

Adaptive Control

Little, John D. C., "Managers and Models: The Concept of a Decision Calculus," Management Science, 16, B466-485 (April, 1970).

A manager tries to put together the various resources under his control into an activity that achieves his objectives. A model of his operation can assist him but probably will not unless it meets certain requirements. A model that is to be used by a manager should be simple, robust, easy to control, adaptive, as complete as possible, and easy to communicate with. By simple is meant easy to understand; by robust, hard to get absurd answers from; by easy to control, that the user knows what input data would be required to produce desired output answers; adaptive means that the model can be adjusted as new information is acquired; completeness implies that important phenomena will be included even if they require judgmental estimates of their effect; and, finally, easy to communicate with means that the manager can quickly and easily change inputs and obtain and understand the outputs.

Such a model consists of a set of numerical procedures for processing data and judgments to assist managerial decision making and so will be called a decision calculus. An example is described.

Markov Models

Morse, Philip M., and Harold J. Yaffe, "A Queuing Model for Car Passing," M.I.T. Operations Research Center Technical Report No. 51, April, 1970.

A model is developed for the flow of automobiles in one direction along a two-lane country road. The model takes into account the fact that cars differ in speed, that slower cars accumulate queues behind them, and that the rate of

Summary of Research Findings (cont'd)

escape from such a queue by passing the lead car depends on the speed of the lead car, on the nature of the road, and on the density of traffic going in the opposite direction. Equations for the stochastic steady state of the system reduce to an integral equation for the mean queue length, as a function of lead-car speed and of a queue-delay parameter. Solutions are obtained, with tables and graphs, for two different assumptions regarding the dependence of passing delay on lead-car speed. In both cases the model exhibits a sudden change from sparse-traffic conditions (where queuing is rare and delay of the faster cars is minimal) to the dense-traffic conditions (where nearly all cars find themselves trapped in a slow queue as the passing-delay parameter is increased beyond its transitional value). Such a sudden "phase change" in traffic character is typical of actual traffic under the specified conditions. Measured dependence of queue length on lead-car speed checks nicely with the model, for sparse-traffic conditions.

Queuing Theory and Markov Models

Mehta, Cyrus R., "Time Sharing with an Explicit Priority Queuing Discipline," M.I.T. Operations Research Center Technical Report No. 54, July, 1970. Adapted from an S. M. Thesis, Sloan School of Management, February, 1970, supervised by Prof. G. M. Kaufman.

A mathematical analysis is given of a round-robin computer time sharing system that is characterized by homogeneous Poisson arrivals, exponentially distributed service times and an ordered priority queue. Each new arrival buys a position in this queue by offering a non-negative payment to the manager of the computer facility. The system is modeled as a continuous time stochastic process and analyzed by an imbedded Markov chain. Relevant queuing statistics are identified and derived in terms of system parameters. An optimization problem in revenue maximization is formulated and solved with two system parameters as decision variables.

Papademos, Lucas D., "Markov Decision Processes and the Analysis of a Queuing System," S. B. Thesis, Department of Physics, June, 1970, supervised by Prof. P. M. Morse.

The basic concepts of Markov decision processes are employed to measure and compare the effectiveness of a queuing system consisting of a facility that accepts units of two different types, each characterized by a different pair of probability density functions for the arrival and service times. Five alternative schemes of servicing units that belong to two different classes have been analyzed.

Summary of Research Findings (cont'd)

Order Statistics

Welsch, Roy E., "Limit Laws for Extreme Order Statistics from Strong-Mixing Processes," M.I.T. Operations Research Center Technical Report No. 52, April, 1970. Submitted for publication in Annals of Mathematical Statistics.

This paper characterizes the possible limit laws for a sequence of normalized extreme order statistics (maximum, second maximum, etc.) from a stationary strong-mixing sequence of random variables. It extends the work of Loynes who considered only the maximum process. The maximum process leads to limit laws that are the same three types that occur when the underlying process is a sequence of independent random variables. The results presented here show that the possible limit laws for the k-th maximum process ($k > 1$) from a strong-mixing sequence form a larger class than can occur in the independent case.

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The reports relevant to the reporting period covered herein are:

1. M.I.T. Operations Research Center Annual Report, July 1, 1965 through June 30, 1966.
2. M.I.T. Operations Research Center Annual Report, July 1, 1966 through June 30, 1967.
3. M.I.T. Operations Research Center Annual Report, July 1, 1967 through June 30, 1968.
4. M.I.T. Operations Research Center Annual Report, July 1, 1968 through June 30, 1969.
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13. ABSTRACT

This project has been concerned with extending the fundamental theories underlying operations research. In particular it has focused on certain areas in mathematical programming, adaptive control, decision theory, and queuing theory. The research in mathematical programming has centered on integer and mixed integer problems. The adaptive control work has dealt with special classes of stochastic decision problems. The multidimensional utility function problem has been studied in decision theory. In queuing theory, which has long been descriptive in its approach, the emphasis has been on normative models, especially Markovian decision processes.

14. KEY WORDS

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